Teachers' Cognitive Functioning in the Context of Questions Using the Arithmetic Mean

Rosemary A. Callingham

University of Tasmania / Education Department of Tasmania

Abstract

The arithmetic mean is probably the most commonly taught statistical measure. This study of 136 pre- and in-service teachers concentrates on responses to questions about the arithmetic mean in different contexts. It was devised to allow for increasingly sophisticated understanding of the concept to be demonstrated in the concrete-symbolic mode, and also allowed for ikonic mode processing. The results are considered from the perspective of cognitive development. Cycles of response in both ikonic and concrete-symbolic mode are identified and described. A theoretical model to explain the interaction between the modes is proposed. The implications of these findings is briefly discussed.

Introduction

It is now widely accepted that chance and data should be a part of the general mathematics curriculum. This has led to a developing interest in the knowledge and skills of teachers required to teach chance and data concepts. Ample evidence exists that there are major misconceptions about data interpretation, and that moves to change this have only been partly successful (eg. Pollatsek, Lima & Well, 1981; Gal, 1992; Cox & Mouw, 1992). Despite this, little work has been undertaken to identify the cognitive processes being used by adults to solve chance and data problems. This paper reports the results of a study of the cognitive functioning of pre- and in-service teachers when given questions which could be solved by using the arithmetic mean. This concept was chosen because it is arguably the most widely taught statistical measure. Assumptions are made that the basic tools of descriptive and inferential statistics, calculations of various measures of central tendency and spread, including the arithmetic mean, are available to teachers, and that they, therefore, have the necessary knowledge required to develop understanding in students. Recent studies by Bright, Berenson and Friel (1993) and Mokros and Russell (1992) apparently contradict this belief.

Methodology

A short questionnaire was designed and administered to 136 pre- and in-service teachers. All four questions related to the arithmetic mean, although the contexts did allow other interpretations of average to be used. The in-service teachers were all undertaking a professional development workshop in mathematics. As such they were not a random group of teachers but, from experience, appeared to be typical of school staffs, having a wide range of background experiences. The pre-service teachers were all final year students from the University of Tasmania. Participation was voluntary, and permission forms were received. Members of both pre- and in-service teacher groups taught all grades from kindergarten to grade 10. On all occasions the questionnaire was administered in a group situation, with no discussion between subjects about the questions. Calculators were freely available. The questions were designed to be hierarchical to some extent, as well as placing the mean in different contexts. Information was presented in ways which, from experience, teachers would recognise, utilising text, tables and graphs as appropriate.

Question 1 involved the straightforward calculation of the average mass of an object given a set of measurements. This was not unlike a typical text book problem. The context was practical - a set of results from an experiment - and one outlier was included.

Questions 2 and 3 were adapted from those used by Mokros and Russell (1992). The information was related to the spelling scores of groups of students and was presented in graphical form. Participants were asked to identify the group having the better spellers. Calculation of the arithmetic mean, using results from the graph was the expected response, although it was recognised that other answers were possible. Question 2 was seen as straightforward, involving two groups of only ten students, one of which was clearly better than the other. Question 3 was essentially the same problem, but with two larger groups having different numbers of students, and a very similar performance.

The final question was a weighted average question, similar to those used by Pollatsek, Lima and Well (1981). The context was finding the average number of babies born using data about birth numbers in small and large hospitals.

The responses obtained were analysed from a developmental perspective using the framework provided by the SOLO Taxonomy (Biggs & Collis, 1982). Using this approach, increasing complexity of response can be recognised by the way in which information is utilised to answer a question. Five levels of response have been identified: prestructural, unistructural, multistructural, relational and extended abstract. These categories form a cycle through which the level of response to any learning situation may be monitored (Campbell *et al.*, 1992). The answers obtained from the survey participants were analysed by considering the types of response to individual questions. The classification of the responses followed the unistructural, multistructural, relational model proposed by Watson *etal* (1993) using the SOLO Taxonomy.

The identification of different modes of thinking of increasing abstraction in which information is handled is well documented (e.g.Biggs & Collis, 1991). Within all of these modes of thinking are cycles of increasing complexity referred to earlier. It has been further hypothesised (Campbell, Watson & Collis, 1992) that within each mode there may be more than one cycle. The ikonic and concrete-symbolic modes are of most interest in the school context, and were specifically targeted in this study.

Results

While the questions were themselves hierarchical, allowing consideration of the demonstrated understanding of the concept of average, it is the cycle of response within each question that is of interest here. The answers given by the participants to the questions were often unexpectedly creative, and varied from an unsophisticated calculation of the mean in a concrete-symbolic mode, to those which appeared to be utilising multimodal functioning. Questions 1 and 4 only accessed concrete-symbolic thinking. On the other hand, questions 2 and 3, which had a graphical presentation, allowed consideration of the ikonic mode and the multi-modal functioning of some teachers. The responses to each question are reported separately.

Question 1.

This question was framed to access concrete-symbolic thinking only. Given the context of the question, a reasonable relational response would have been to discuss the inclusion of the outlier. No response showed this thinking, the most complex being classified as multistructural. This may indicate that the practice of using a number of measurements obtained from an experiment, critically examining them and then using the appropriate summative measure is not widespread.

Question 4

This question also accessed the concrete-symbolic mode, but required a higher level of thinking than the response to question 1. The answers to this question were again relatively clear cut. The only successful strategy seen was again multistructural in approach, with the calculation completed in a stepwise manner, the total number of babies being calculated first and then the average obtained from this.

For these two questions requiring different levels of response to the concept of the arithmetic mean, utilising that concept to solve a problem led to a response which was at best multistructural. The answers given typically used a stepwise approach to the calculation. No respondent demonstrated a sophisticated, integrated approach to the necessary calculations.

Questions 2 and 3.

These two questions concerned performance on a spelling test of different groups of students. They differed from the other two in that a visual component was present as the information was presented in graph form. A number of alternative, and unexpected solutions was demonstrated.

Question 2 was designed so that the difference between the two groups was immediately obvious visually. The groups were small, and of the same size, so that a direct comparison was easy and valid. Only sixteen teachers out of the sample of 136 (11.8%) used a visual comparison only. Nearly half (58 teachers, 41.2%) used the arithmetic mean to justify their answers. A surprisingly large number (35 teachers, 25.7%) used some sort of numerical system other than recognised statistical measures. These strategies included a consideration of the range of the data, the total number of questions that the group got correct or the numbers of students getting more correct than some arbitrary pass mark. This latter strategy was more prevalent among high school teachers.

Question 3 was more complex. The difference between the groups was very small and not obvious visually. In addition, group sizes were much bigger than those in the previous question and differed considerably. The proportion of teachers responding visually did not differ greatly from that in question 2 (17 teachers out of 131, 13.0%). Similar results were also shown for the use of the arithmetic mean (58 teachers, 44.3%). In contrast, numerical methods other than recognised statistical measures were preferred by only fifteen teachers (11.5%), less than half of those using these strategies for question 2. Some possible reasons for this difference will now be considered.

Question 2 effectively invited a response in the ikonic mode. It provided a very clear visual picture of the information, which could be simply compared. Teachers are also familiar with the context - deciding which group has better spellers. This context, however, invites justification from teachers. They work with the necessity to explain and justify judgements about individuals and groups of students, and in their day to day operations spend considerable time collecting evidence and records in order to do this. For teachers, therefore, these questions could produce some conflict between the visual presentation accessing an ikonic mode of thinking, and the need to explain and justify judgements of this nature in the concrete-symbolic mode.

It seems possible that the changeover seen from ikonic to concrete-symbolic thinking demonstrates a parallel structure of modes of thinking, hypothesised by Watson *et al.*. (1993). Multi-modal thinking, particularly when faced with a novel problem, has been reported by a number of researchers (eg. Collis & Romberg, 1990; Watson, Campbell & Collis, 1993). Essentially, when faced with a problem, people select that mode of thinking which is most appropriate for them in that situation. While sensori-motor functioning is available, in most school-based circumstances the real choice is between ikonic and concrete-symbolic thought. The problem may then be followed through in the chosen mode, or there may be transfer from one mode to the other. This may be represented diagrammatically as shown in figure 1.

Following this pathway, teachers faced with a straightforward graphical problem, such as that in question 2, could choose the ikonic (IK) route, or the concrete-symbolic (CS) path. An individual choosing the ikonic path then has a choice of two possible paths, labelled (i) and (ii), for processing the problem. The choice of the IK(ii) route allows transfer into the concrete-symbolic mode at points B and C. Given the context of the question, spelling scores, the use of "work place" mathematics to process the problem, and transfer into the concrete symbolic mode to justify and communicate the answer seems very appropriate for teachers.

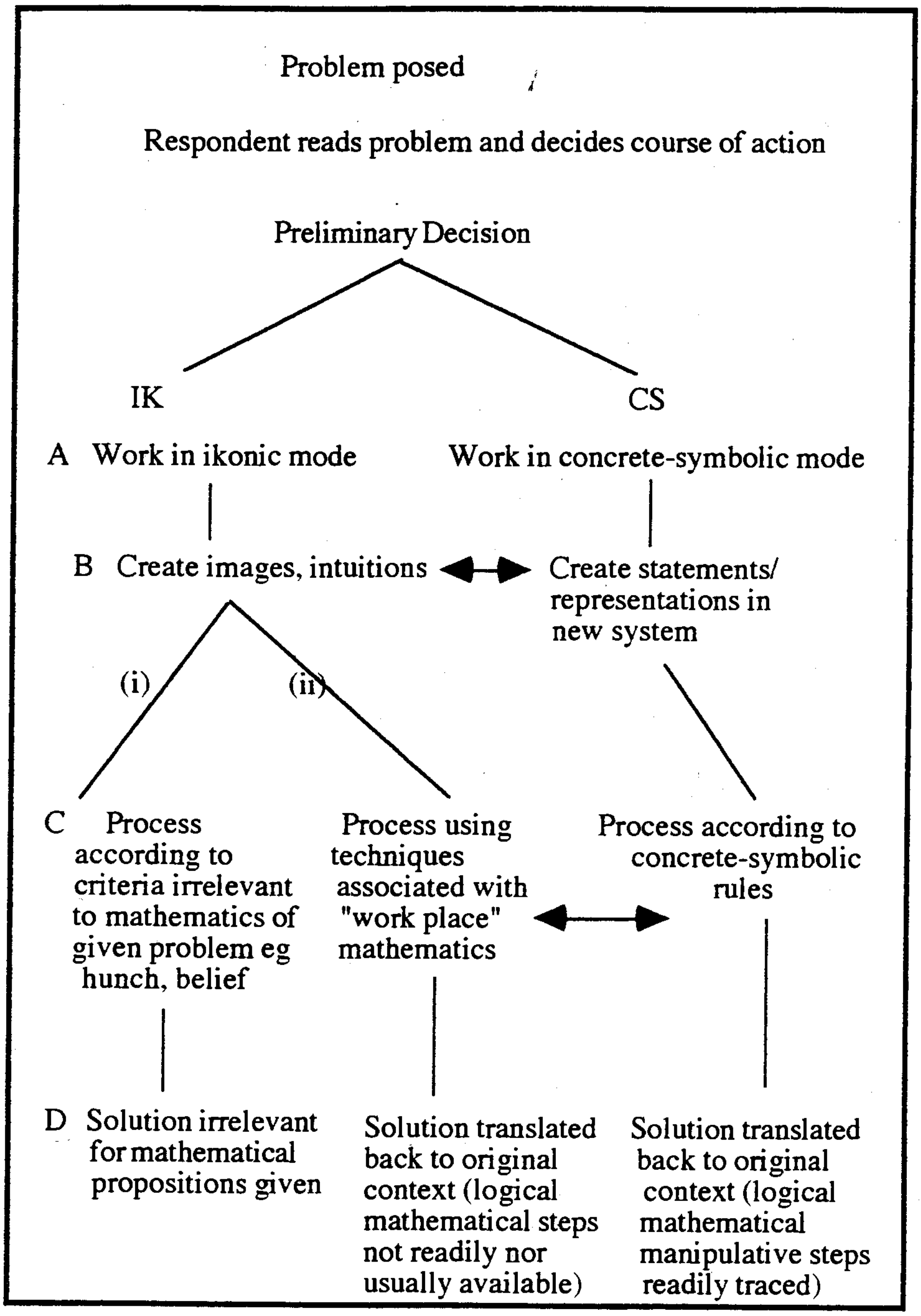

Figure 1 The Problem Solving Path (from Watson, Campbell & Collis, 1993)

When faced with a more complex ikonic problem, teachers seemed to prefer to move down the concrete-symbolic path. High school teachers particularly changed their strategy from the easier problem of question 2 to the more complex problem of question 3. This was not so evident for primary school teachers who showed no significant difference in their responses to these questions.

This apparent choice of concrete-symbolic reasoning may be supported by ikonic processing since movement between the modes is possible at different points in the problem solving path.

Using this model it seems possible that the generalised categories of response to questions 2 and 3 could be mapped onto a problem solving path in either ikonic or concrete-symbolic modes. One possible mapping is shown in figure 2. The categories IK_A, IK_B and IK_C refer to the levels of ikonic response indicated in figure 1. Campbell *et al*. (1992) hypothesised that developmental sequences in the concrete-symbolic mode could be described as cycles of increasingly complex responses. The responses obtained here may also demonstrate a developmental sequence in the ikonic mode. Thus the levels IK_A, IK_B and IK_C could be considered as equivalent to unistructural, multi-structural and relational SOLO taxonomy levels. The U, M and R categories under concrete-symbolic refer to the SOLO taxonomy levels referred to earlier.

Type of response	SOLO level of response Ikonic	 Concrete-symbolic
Intuition leading to incorrect response	IK_A	Pre-structural
Visual strategy only	IK_A	U
"Counting" strategy	IK_B	M
"Pass mark" strategy	IK_C	M
Use of Mean	No ikonic support	R

Figure 2. Summary of types of response to survey questions 2 and 3

The first two types of responses, incorrect and visual only, were categorised as a unistructural ikonic response, IK_A, since no attempt had been made in the ikonic mode to process the information by considering the various features of the graph. Incorrect responses were mainly those which used no concrete-symbolic reasoning, or made no attempt to solve the problem, typified by comments such as "too difficult". Those classified as using a visual strategy were those which referred only to visual features in the justification. This was classified as a unistructural concrete-symbolic response. The "counting" strategy followed the ikonic pathway to IK_B initially, since these responses mainly focussed on some particular feature of the graph, such as the spread or range of the data. This was seen as a multistructural response in the concrete-symbolic mode since this ikonic information was translated into a concrete-symbolic justification which included some sort of numerical processing. Using the "pass mark" was classified as a higher level ikonic response, IK_C, since work place mathematics was involved. In the concrete-symbolic mode it was classified as multistructural since the same level of reasoning was employed as the "counting" strategy. Use of the mean was seen as the top level of concrete-symbolic reasoning in this problem solving cycle. Teachers using this method, especially in question 3, did not appear to rely on ikonic support.

Responses obtained to these questions seemed to justify this model. A relational ikonic response to question 2 would take into account the "look" of the data, the shape and spread shown on the graph, the number of students in the group and the level of their response, compare the two groups, and integrate this into a judgement of which group is better. The necessity to justify this would then lead to translation of this judgement into concrete-symbolic mode. Teachers appeared to prefer to use some numerical basis for this. Many went to the mean for corroboration of their intuitive findings, but others, in particular experienced high school teachers, used an arbitrary "pass mark" to support their findings. This approach is consistent with the everyday working tools which teachers use. In SOLO terms the concrete-symbolic responses were rarely more than multi-structural. This again is consistent with the context, since teachers are looking only for an acceptable explanation of their intuitive understanding, rather than a consideration of the "best" method for comparing the groups.

Question 3 was more problematic for teachers. It accessed the ikonic mode through its presentation in graphical form, but was considerably more complex than the earlier question. Nevertheless, a number of teachers responded with a visual justification only. The biggest difference in response from the previous question was in the type of numerical justification utilised. Many of those teachers who had used other numerical methods, in particular the arbitrary pass mark, went straight to the mean when faced with this more difficult problem. This was particularly noticeable in high school teachers. Using the mean as opposed to less complex numerical methods could be seen as a higher level concrete-symbolic response. A concrete-symbolic mode of operating seemed to be preferred when faced with a problem which was not easily solved in the ikonic mode. No reasons were given for the changed approaches. Teachers, rather, seemed to be choosing the method which seemed most appropriate to them under the circumstances.

Implications

If the multi-modal functioning which is apparent in this study is a common phenomenon, then attention should be paid to developing the skills of moving from one mode of thinking to another. This would allow for interaction of the two modes, complementing and reinforcing each other.

One implication of this may be that the more content knowledge teachers have, the less they need to concentrate on the subject matter. They can use ikonic thinking processes to deal with this, which frees the concrete-symbolic mode for different tasks. These tasks could include applying appropriate pedagogic knowledge to student learning. From personal experience, both in the classroom and in work with other teachers, experienced teachers commonly react to children's queries about content at an intuitive level. They can look at a piece of work presented by a child and recognise whether the answer is correct or not without actually processing it. Having considered the work in this way, the teacher can then pose suitable questions to help the child refine understanding. While this is only a conjecture, this study provides some evidence for multi-modal functioning in teachers which is worthy of further consideration. If the multi-modal thinking identified here is confirmed in other areas, then opportunities need to be provided in teacher development and pre-service courses for teachers to experience a variety of modes of thinking, in the same way that multi-modal approaches are now being recommended for children's learning (Biggs & Collis, 1991).

No generalisations can be made from a study of this size and scope, but further research into the psychology of teaching could provide more information about this area, and make recommendations for pre- and in-service teacher development.

References

Biggs, J.B. & Collis, K.F. (1982). Evaluating the quality of learning: the SOLO taxonomy. New York: Academic Press.

Biggs, J.B. & Collis, K.F. (1991). Multi-modal learning and the quality of intelligent behaviour. In H. Rowe (Ed.). Intelligence: Reconceptualisation and measurement. Hillsdale, N.J.: Lawrence Erlbaum.

Bright, G.W.; Berenson, S.B. & Friel, S. (1993). Teachers' knowledge of statistics pedagogy. Paper presented at the annual meeting of the Research Council for Diagnostic and Prescriptive Mathematics, Melbourne, FL.

Campbell, K.J., Watson, J.M. & Collis, K.F. (1992). Volume measurement and intellectual development. Journal of Structural Learning, 11, 3, 279-298.

Collis, K.F. & Biggs, J.B. (1991). Developmental determinants of qualitative aspects of school learning. In G.T. Evans (Ed.). Learning and teaching cognitive skills. Melbourne:A.C.E.R.

Collis, K.F. & Romberg, T.A. (1990). "The standards": Theme and assessment. In K. Milton & H. McCann (Eds.), Mathematical Turning Points - Strategies for the 1990's. Hobart: A.A.M.T.

Cox, C., & Mouw, J.T.(1992). Disruption of the representativeness heuristic: Can we be perturbed into using correct probabilistic reasoning? Educational Studies in Mathematics, 23,163-178.

Gal, I., (1992). Reaching out: Some issues and dilemmas in expanding statistics education. Proceedings of the August 1992 International Statistics Institute Roundtable on Teaching Data-analysis, Bishop University, Canada.

Mokros, J.R., & Russell, S.J. (1992). Children's concepts of average and representativeness (Working Paper 4-92). Cambridge, MA: TERC.

Pollatsek, A.,Lima, S., & Well, A.D., (1981). Concept or computation: Students understanding of the mean. Educational Studies in Mathematics, 12, 191-204.

Watson, J.M.,Campbell, K.J. & Collis, K.F. (1993). Multimodal functioning in understanding fractions. Journal of Mathematical Behaviour, 12, 45-62.